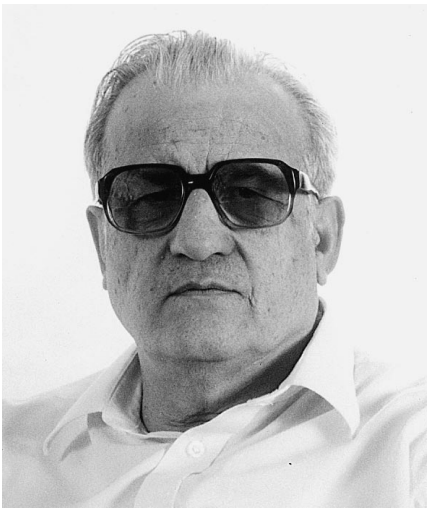

Waleed Al-Salam
1926–1996

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In Memoriam

Waleed Al-Salam

July 15, 1926–April 13, 1996

Waleed Al-Salam, professor emeritus at the University of Alberta, died on April 13, 1996, at the age of 69. Born in Baghdad, Iraq, on July 15, 1926, Waleed studied at the University of California, Berkeley, where he received a bachelor's degree in engineering physics in 1950 and an M.A. in mathematics in 1951. Following a few years of teaching at the University of Baghdad, he resumed his graduate studies in mathematics at Duke University. He wrote a dissertation on the Bessel polynomials under Leonard Carlitz and received his Ph.D. in 1958. He then held positions at the University of Baghdad, Texas Technological University, and the University of Alberta at Calgary (now called the University of Calgary) before accepting a position at the University of Alberta in Edmonton. He remained at Edmonton with brief visits to Cambridge University, the American University of Beirut, Arizona State University, and the University of South Florida until his retirement in 1992.

Edmonton was an especially lively place in the late 1960s and the 1970s and Waleed and the many visitors he brought to Edmonton were a major part of the life there. A leading figure in special functions, Waleed's early work was a big influence in the resurgence of interest in orthogonal polynomials that began in earnest in the 1960s. This was especially true of his work with "q-polynomials," which identified markers and found clues to the rich underlying theory. Some of Waleed's contributions to this subject are cited in the current bible on q-series, the book by Gasper and Rahman [6].

His 1965 paper with Carlitz [A42] introduced one-parameter generalizations of the discrete Hermite polynomials. (A reference starting with A refers to Waleed's publication list while the others refer to references at the immediate end of the text.) These polynomials, denoted by $U_n^{(a)}(x)$ and $V_n^{(a)}(x)$ and known as the Al-Salam–Carlitz polynomials are, from a combinatorial perspective, the q-analogues of the Charlier polynomials.

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Al-Salam and Carlitz gave many identities and relations for their polynomials and, in particular, gave their orthogonality relations explicitly. The moment problem associated with $V_n^{(a)}(x)$ is indeterminate in certain cases, and for some of these cases, the distribution functions found by Al-Salam and Carlitz are N-extremal solutions, the first explicitly known examples of such. The corresponding Hamburger and Stieltjes moment problems were later completely solved by Berg and Valent [3].

Another topic on which Waleed wrote extensively, including the very informative survey article [A84], was characterization theorems. In his early work, he gave several characterizations of some of the classical orthogonal polynomials. His paper with Chihara [A56] characterized the Jacobi, Hermite, and Laguerre polynomials as the only orthogonal polynomials satisfying a differential–difference equation of a certain simple form. This confirmed the first of three conjectures made by Karlin and Szegő in their treatise on determinants of orthogonal polynomials [9]. (The second conjecture has recently been settled in the negative by Kim and Kwon [10] while the third one was already known by the well-known result of Hahn [8].) Askey then posed a generalization of the problem which was then solved by Maroni [11] who coined the label "semi-classical" for the resulting orthogonal polynomials. An even more general characterization problem was later solved by Bonan and Nevai [4]. Waleed's last published paper [A86] characterized the continuous q-Hermite polynomials as being the only orthogonal polynomials which are q-Appell, that is, have degenerating function of the form $A(t)\, \mathscr{E}_q(xt; a, b)$, $\mathscr{E}_q$ being the q-exponential function on a q-quadratic grid.

Al-Salam and Chihara [A62] characterized pairs of orthogonal polynomials $\{r_n(x)\}$ and $\{s_n(x)\}$ for which $\sum_{k=0}^{n} r_k(y)\, s_{n-k}(x)$ are orthogonal in x for infinitely many y's. This class contains some of the classical polynomials such as Hermite and Laguerre polynomials and contains one new and interesting set of orthogonal polynomials which became known as the Al-Salam–Chihara polynomials. Their weight function was found later by Askey and Ismail [1] and Askey and Wilson [2]. It was also realized that these polynomials are part of a larger scheme of q-orthogonal polynomials in which the Askey–Wilson polynomials sit at the top of the scheme and the Al-Salam–Chihara polynomials play the role of the Laguerre polynomials in the $q = 1$ scheme.

A seminal paper Waleed wrote with Allaway and Askey is [A77], which introduced the sieved ultraspherical polynomials. They were first mentioned in [A76], by the same authors. The weight function of these polynomials vanishes at isolated points in the interval of orthogonality and later authors managed to use polynomials of a similar type to explicitly get polynomials orthogonal on several intervals. The sieved ultraspherical polynomials were also used to construct polynomials whose spectrum has

imbeded discrete masses [7] and to construct a class of discrete measures on $[-1, 1]$ for which the orthogonal polynomials have converging recurrence coefficients [12]. A sample of the structural properties of these polynomials is in [5].

Waleed also worked on a variety of other topics. He wrote a well-known paper on the Bessel polynomials [A8; see also A6, A22], derived several Turán type inequalities, established generating functions for various polynomials and functions, and investigated other aspects of special functions. He contributed to the theory of q-fractional integrals and derivatives which he introduced in [A48]. He proved a Leibniz rule for them in his joint work with Verma [A58]. The resulting operators were used in [A64] to construct reproducing kernels for the little q-Jacobi polynomials.

He also worked on q-beta integrals and biorthogonal polynomials and functions jointly with Verma and with Ismail. In particular, [A85] contains an evaluation of a q-beta integral and introduces a system of rational functions biorthogonal on the unit circle. These functions are unit circle analogues of the Askey–Wilson polynomials. They also are prototypes of the later theory of continued R-fractions.

A valuable service Waleed provided to the special functions community was to establish an electronic repository for papers on special functions, which he instituted upon his retirement in 1992. He had the vision to realize the importance of this endeavor and he spent considerable time and energy in creating the system, which has become an indispensable source of information on the subject. Waleed maintained the system until 1995, when his failing health caused him to pass on the task to Hans Haubold in Vienna.

Waleed is survived by his wife and frequent collaborator, Nadhla (née Abdul-Halim), four sons, Nabil, Ramsey, Haytham, and Yaseen, two daughters, Leila and Salma, four mathematical sons, Iftikhar Ahmad (M.Sc. 1969), William Allaway (Ph.D. 1972), M. N. Bajaj (M.Sc. 1969), and Mourad Ismail (Ph.D. 1974), and seven mathematical grandchildren.

Both writers of this note visited Waleed frequently in Edmonton and collaborated with him regularly. He was always a gracious, friendly, and generous host, as all of his many visitors could attest. Our collaborations with him were always interesting and enjoyable.

We miss you, Waleed.

REFERENCES

1. R. Askey and M. E. H. Ismail, Recurrence relations, continued fractions and orthogonal polynomials, *Mem. Amer. Math. Soc.* **300** (1984).
2. R. Askey and J. A. Wilson, A set of hypergeometric polynomials that generalize Jacobi polynomials, *Mem. Amer. Math. Soc.* **319** (1985).

3. C. Berg and G. Valent, The Nevanlinna parameterization for some indeterminate Stieltjes moment problems associated with birth and death processes, *Methods Appl. Anal.* **1** (1994), 169–209.
4. S. Bonan and P. Nevai, Orthogonal polynomials and their derivatives, I, *J. Approx. Theory* **40** (1984), 134–147.
5. J. Charris and M. E. H. Ismail, On sieved orthogonal polynomials VII: Generalized polynomial mappings, *Trans. Amer. Math. Soc.* **340** (1993), 71–93.
6. G. Gasper and M. Rahman, "Basic Hypergeometric Series," Cambridge Univ. Press, Cambridge, UK, 1990.
7. J. Geronimo and W. Van Assche, Orthogonal polynomials on several intervals via a polynomial mapping, *Trans. Amer. Math. Soc.* **308** (1988), 559–581.
8. W. Hahn, Über die Jacobischen Polynome und zwei verwandte Polynomklassen, *Math. Z.* **39** (1935), 634–638.
9. S. Karlin and G. Szegő, On certain determinants whose elements are orthogonal polynomials, *J. Anal. Math.* **8** (1960), 1–157.
10. D. H. Kim and K. H. Kwon, On a conjecture by Karlin and Szegő, *Proc. Amer. Math. Soc.* **124** (1996), 227–231.
11. P. Maroni, Une caractérisation des polynômes orthogonaux semi-classiques, *C. R. Acad. Sci. Paris Sér. I Math.* **301** (1985), 269–272.
12. W. Van Assche and A. P. Magnus, Sieved orthogonal polynomials and discrete measures with jumps dense in an interval, *Proc. Amer. Math. Soc.* **106** (1989), 163–173.

Publications of Waleed A. Al-Salam

1. Some relations involving the Jacobi polynomials, *Portugal. Math.* **15** (1956), 73–77.
2. On the product of two Legendre polynomials, *Math. Scand.* **4** (1956), 239–242.
3. On a characterization of some orthogonal functions, *Amer. Math. Monthly* **64** (1957), 29–32.
4. Some remarks on the Turán expressi on, *Bull. College Sci.* (*Baghdad*) **2** (1957), 104–111.
5. On a generalized Hermite polynomial, *Boll. Un. Mat. Ital.* **12** (1957), 241–246.
6. On the Bessel polynomials, *Boll. Un. Mat. Ital.* (3) **12** (1957), 227–229.
7. On a characterization of orthogonality, *Math. Magazine* **31** (1957), 41–44.
8. The Bessel polynomials, *Duke Math. J.* **24** (1957), 529–546.
9. Note on a q-identity, *Math. Scand.* **5** (1957), 202–220.
10. A generalization of some polynomials related to the theta functions, *Rev. Math. Univ. Parma* **8** (1957), 381–395.
11. (with L. Carlitz), Some finite summation formulas for the classical orthogonal polynomials, *Rend. Mat. Appl.* (*Roma*) (5) **16** (1957), 74–95.

12. (with L. Carlitz), A q-analog of a formula of Toscano, *Boll. Un. Mat. Ital.* (3) **12** (1957), 414–417.
13. (with L. Carlitz), Generalized Turán expressions for certain hypergeometric series, *Portugal. Math.* **16** (1957), 119–127.
14. (with L. Carlitz), The expansion of certain products containing Bessel functions, *Le Matematiche* **12** (1957), 31–34.
15. On some generating functions for the product of two Jacobi polynomials, *Rev. Mat. Hisp.-Amer.* (*Madrid*) (4) **18** (1958), 1–5.
16. On some theorems on permutations, *Amer. Math. Monthly* **65** (1958), 615–616.
17. Some generating functions for the Laguerre polynomials, *Portugal. Math.* **17** (1958), 49–52.
18. (with L. Carlitz), Congruence properties of the classical orthogonal polynomials, *Duke Math. J.* **25** (1958), 1–10.
19. (with L. Carlitz), Finite summation formulas and congruences for Legendre and Jacobi polynomials, *Monatsh. Math.* **62** (1958), 108–118.
20. (with L. Carlitz), Bessel polynomials and Bernoulli numbers, *Arch. Math.* **9** (1958), 412–415.
21. q-Bernoulli numbers and polynomials, *Math. Nachr.* **17** (1959), 239–260.
22. Some functions related to the Bessel polynomials, *Duke Math. J.* **26** (1959), 519–540.
23. On the Turán inequality for certain polynomials, *Amer. Math. Monthly* **66** (1959), 46–49.
24. (with L. Carlitz), Bernoulli numbers and Bessel polynomials, *Duke Math. J.* **26** (1959), 437–446.
25. (with L. Carlitz), Some determinants of Bernoulli, Euler, and related numbers, *Portugal. Math.* **18** (1959), 91–99.
26. Some integral formulas for certain Turán expressions, *Bull. Coll. Sci.* (*Baghdad*) **5** (1960), 19–21.
27. Generating functions for certain Turán expressions, *Rev. Mat. Fis. Teor.* (*Tucuman*) **13** (1960), 85–93.
28. A generalized Turán expression for the Bessel functions, *Amer. Math. Monthly* **68** (1961), 146–149.
29. (with L. Carlitz), The expansion in the product of Bessel functions, *Portugal. Math.* **22** (1963), 153–160.
30. (with L. Carlitz), The Gegenbauer addition theorem, *J. Math. Phys.* **42** (1963), 147–156.
31. (with L. Carlitz), Some functions associated with Bessel functions, *J. Math. Mech.* **12** (1963), 911–934; *J. Math. Mech.* **15** (1966), 711–712.
32. (with N. Abdul-Halim), Double Euler Transformation of certain hypergeometric functions, *Duke Math. J.* **30** (1963), 51–62.

33. Some characterizations of the Laguerre and Hermite polynomials, *Michigan Math. J.* **10** (1963), 381–383.
34. Operational derivation of some formulas for the Hermite and Laguerre polynomials, *Boll. Un. Mat. Ital.* (3) **18** (1963), 381–383.
35. (with N. Abdul-Halim), A characterization of the Laguerre polynomials, *Sem. Mat. Univ. Parma* **34** (1964), 176–179.
36. (with L. Carlitz), Bateman's expansion for the product of two Bessel functions, *Ann. Mat. Pura Appl. Ser.* (4) **65** (1964), 97–112.
37. Operational representation for the Laguerre and Hermite polynomials, *Duke Math. J.* **31** (1964), 127–142.
38. (with L. Carlitz), An addition theorem for the confluent hypergeometric series, *Portugal. Math.* **25** (1964), 95–101.
39. On the orthogonality of some system of orthogonal polynomials, *Collect. Math.* **16** (1964), 187–192.
40. On the characterization of a set of orthogonal polynomials, *Boll. Un. Mat. Ital.* **19** (1964), 448–450.
41. Saalschützian theorems for basic double series, *J. London Math. Soc.* **40** (1965), 455–458.
42. (with L. Carlitz), Some orthogonal q-polynomials, *Math. Machr.* **30** (1965), 47–61.
43. Some remarks on some operational formulas, *Sem. Mat. Univ. Padova* **35** (1965), 128–131.
44. (with L. Carlitz), A note on the Hardy–Hille formulas, *Proc. Glasgow Math. Assoc.* **7** (1965), 55–60.
45. Characterization of certain classes of orthogonal polynomials related to elliptic functions, *Ann. Mat. Pura Appl. Ser.* 4 **67** (1965), 75–94.
46. (with L. Carlitz), On certain generating functions for the Hermite polynomials, *Portugal. Math.* **25** (1966), 35–45.
47. On a characterization of Meixner's polynomials, *Quart. J. Math. (Oxford) Ser.* (2) **17** (1966), 7–10.
48. q-analogues of Cauchy's formula, *Proc. Amer. Math. Soc.* **17** (1966), 616–621.
49. Some fractional q-integrals and q-derivatives, *Proc. Edinburgh Math. Soc.* **15** (1966), 134–140.
50. q-Appell polynomials, *Ann. Mat. Pura Appl. Ser.* (4) **77** (1967), 31–46.
51. (with N. A. Al-Salam), Some characterizations of the ultraspherical polynomials, *Can. Math. Bull.* **11** (1968), 457–464.
52. (with A. Verma), Some orthogonality preserving operators, *Proc. Amer. Math. Soc.* **23** (1969), 136–139.
53. (with A. Verma), Generalized Sheffer polynomials, *Duke Math. J.* **37** (1970), 361–365.
54. (with A. Verma), Fibonacci numbers and Eulerian polynomials, *Fibonacci Quart.* **9** (1971), 18–22.

55. A note on the Hermite polynomials, *Bull. Inst. Politehn. Iasi Sect. Mat. Mec. Teor. Fiz.* **17** (1971), 81–90.
56. (with T. S. Chihara), Another characterization of the classical orthogonal polynomials, *SIAM J. Math. Anal.* **3** (1972), 65–70.
57. (with M. E. H. Ismail), Some operational formulas, *J. Math. Anal. Appl.* **51** (1975), 208–218.
58. (with A. Verma), A fractional Leibniz q-formula, *Pacific J. Math.* **60** (1975), 1–9.
59. (with A. Verma), Remarks on fractional q-integrals, *Bull. Soc. Roy. Sci. Liege* **44** (1975), 600–607.
60. (with A. Verma), Orthogonality preserving operators, I, *Rend. Acad. Naz. Lincei Clas. Sci. Fis. Mat. Natur.* **58** (1975), 833–838.
61. (with A. Verma), Orthogonality preserving operators, II, *Rend. Acad. Naz. Lincei Clas. Sci. Fis. Mat. Natur.* **59** (1976), 26–31.
62. (with T. S. Chihara), Convolutions of orthogonal polynomials, *SIAM J. Math. Anal.* **7** (1976), 16–28.
63. (with M. E. H. Ismail), Polynomials orthogonal with respect to convolution, *J. Math. Anal. Appl.* **55** (1976), 125–139.
64. (with M. E. H. Ismail), Reproducing kernels for the q-Jacobi polynomials, *Proc. Amer. Math. Soc.* **67** (1977), 105–110.
65. (with M. E. H. Ismail), A family of operational calculi, *Math. Japon.* **22** (1978), 571–583.
66. (with N. A. Al-Salam), The centralizer of the Laguerre polynomial set, *Linear Multilinear Algebra* **7** (1979), 149–154.
67. (with N. A. Al-Salam), On a functional equation, *Bull. Can. Math. Soc.* **22** (1979), 235–237.
68. (with T. S; Chihara), On Reimer's recurrences, *Portugal. Math.* **38** (1979), 45–58.
69. (with A. Verma), On an Orthogonal polynomial set, *Indag. Math.* **44** (1982), 334–340; also in *Proc. Konink. Nederl. Akad. Wetensch. Ser. A Math. Sci.* **85**, No. 3, 334–340.
70. (with A. Verma), Some remarks on q-Beta integrals, *Proc. Amer. Math. Soc.* **85** (1982), 360–362.
71. (with A. Verma), q-Konhauser polynomials, *Pacific J. Math.* **108** (1983), 1–7.
72. (with A. Verma), q-Analogs of some biorthogonal functions, *Can. Math. Bull.* **26** (1983), 225–227.
73. (with A. Verma), A pair of biorthogonal sets of polynomials, *Rocky Mountain J. Math.* **13** (1983), 273–279.
74. (with M. E. H. Ismail), Orthogonal polynomials associated with the Roger–Ramanujan continued fraction, *Pacific J. Math.* **104** (1983), 269–283.
75. (with A. Verma), On quadratic transformations of basic series, *SIAM J. Math. Anal.* **15** (1984), 414–421.

76. (with W. Allaway and R. Askey), A characterization of the continuous q-ultraspherical polynomials, *Can. Math. Bull.* **27** (1984), 329–336.
77. (with W. Allaway and R. Askey), Sieved ultraspherical polynomials, *Trans. Amer. Math. Soc.* **284** (1984), 39–55.
78. (with N. Al-Salam), The centralizer of the Laguerre polynomial set, *Rocky Mountain J. Math.* **14** (1984), 713–719.
79. (with A. Verma), Some sets of orthogonal polynomials, *Rev. Tecn. Fac. Ingr. Univ. Zulia* **9** (1986), 83–88.
80. (with T. S. Chihara), q-Pollaczek polynomials and a conjecture of Andrews and Askey, *SIAM J. Math. Anal.* **18** (1987), 228–242.
81. (with J. L. Fields), A differential difference equation, *SIAM J. Math. Anal.* **18** (1987), 873–879.
82. (with M. E. H. Ismail), q-Beta integrals and q-Hermite polynomials, *Pacific J. Math.* **135** (1988), 209–221.
83. (with A. Verma), On the Geronimus polynomial sets, *in* "Proceedings of the Second International Symposium on Orthogonal Polynomials and Their Applications, Segovia 1986," Lecture Notes in Mathematics, Vol. 1329, pp. 193–202, Springer-Verlag, Berlin/New York, 1988.
84. "Characterization Theorems for Orthogonal Polynomials," NATO ASI Series, Vol. 294, pp. 1–24, Kluwer Academic, Dordrecht, 1990.
85. (with M. E. H. Ismail), A q-Beta integral on the unit circle and some biorthogonal rational function, *Proc. Amer. Math. Soc.* **121** (1994), 553–561.
86. A characterization of the Rogers q-Hermite polynomials, *Int. J. Math. Math. Sci.* **18** (1995), 641–648.

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